An Application of Adaptive Control to a Continuous Stirred Tank Reactor

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This paper reports on an investigation of the effect of adaptive control on a closed-loop chemical plant. The plant controlled was a simple closed-loop feedback system, the elements of which were a proportional controller and an elementary continuous stirred-tank reactor. The reaction involved was an exothermic, single-reactant decomposition, with the control criterion being the integral of the square of the deviation of the output composition from a value determined by a reference model. The adaptive control system incorporated an automatic identification scheme and a decision process, and operated in the presence of disturbances in cooling water temperature and/or catalyst activity.

The adaptive control system yielded excellent results. For all disturbances, the output of the adaptive controlled plant remained much closer to the reference model output than did the output of the simply controlled plant. Inclusion of an adaptive capability in a control system thus appears to be desirable and often feasible.

In 1955 automatic control systems technology had reached a plateau in its development. Basic techniques, such as Nyquist, Bode, and root-locus plots, were in universal use in the analysis of linear systems, and describing functions and phase-plane techniques were being widely applied in advanced work on nonlinear systems. It was about this time that a demand for more sophisticated control theory arose, due mainly to the advent of the space age. Conventional theory, which had sufficed until then, was not a powerful enough tool with which to design control systems which had to operate in widely varying and/or unknown environments. More complex theory became a necessity, and thus the field of adaptive control systems was born.

The terms *adaptive control, self-optimizing control,* and many others have been used often in the controls literature during the past few years (*1*, *2*). The ambiguity involved in their usage stems from the lack of a common concept of adaptive control among the researchers in the field; that is, certain systems would be considered adaptive by some, but not by others. Many definitions ranging from the specific to the very general, have been advanced for adaptive control by workers such as Truxal (*3*), Gibson (*4*), and Aris (*5*). In this work an adaptive control system means a control system which, upon sensing large changes within the system or in the surroundings, can vary itself to compensate for these disturbances in order to maintain a good quality of control with respect to certain performance criteria.

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Perhaps the concept can be best understood by an example. Consider a man driving an automobile on a dry highway under sunny skies. Upon seeing the vehicle in front of him stop, the driver applies control action in the form of pressure on the brake pedal. He is an ordinary control element in the sense that his eyes "feed back" information to his brain, such as how much distance remains between his car and the obstacle, and his brain interprets the information into an "input signal" to his foot. Now, consider the same man driving the same automobile over the same highway during a snowstorm. Under this different condition, the driver becomes an adaptive controller. He now may have to apply a very different control action in order to achieve the same result of stopping a few feet from the obstacle. That is, he must "adapt" his control action to fit the new environment. He may even have to apply some test actions, such as short bursts of braking power, to determine what his control action for optimum output will be when the time comes to apply it.

A feedback control system consisting of a continuous stirred tank reactor and a control element can be considered analogous to the man-automobile system. The controller parameters are set such that the closed-loop system, which can be referred to as the plant, responds as desired to variations in an input signal. The overall plant is analogous to the man-automobile system, and the controller can be likened to the driver.

If a reactor parameter or disturbance on which the controller has no influence (such as heat transfer coefficient between cooling water and reactor contents) varies from its original value, the plant will no longer respond as de-

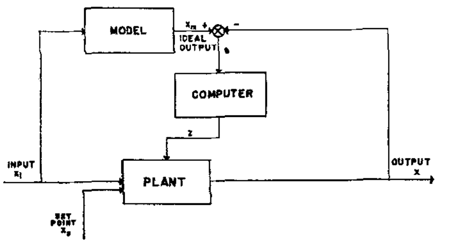

Fig. 1. Model reference plant—adaptive control system.

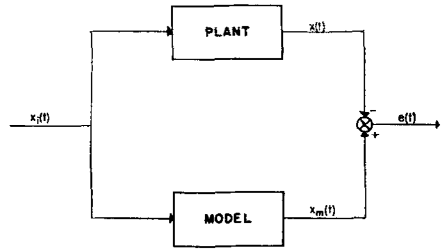

Fig. 2. Basic model reference configuration.

sired to variations in the input signal of importance. That is, the controller parameters are no longer set at their optimal values. Thus, under the new conditions affecting the system, an adaptive controller which could change its own parameters to new optimal values, much as the driver changes his anticipated braking action, would be necessary for continued satisfactory response. The adaptive controller might be one capable of varying the input signal slightly to see how the plant reacts, and on the basis of these test actions, modifying its own parameters to maintain the desired overall dynamic system performance.

Control System

The type of control system studied in this work was a model-reference, plant-adaptive, control system applied to a closed-loop plant which included a continuous stirred tank reactor and a control element. A plant adaptive control system is defined as a system in which the plant input and output signals and parameters are periodically sampled, and then to achieve the desired output, one or more of the plant parameters is adjusted to keep the difference between the plant output and the model output at a minimum (*6*).

Figure 1 embodies the important principles of a model reference, plant-adaptive control system (*7*). In this figure, the basic control system (the plant) represents a single-loop feedback configuration consisting of the process to be controlled and a controller. The adaptive system is then superimposed on the basic operating feedback configuration (the plant) in order to achieve improved performance. Adaptivity is included by comparing a model output $x_m(t)$ with the plant output $x(t)$ to obtain an error $e(t)$, which is a measure of the variation of process dynamics. The model block can be chosen arbitrarily by the designer, and $e(t)$ then becomes a measure of the deviation of the system performance from a specified behavior. The error is operated on by a computer, yielding the signal Z, which adjusts certain plant parameters to bring $e(t)$ toward zero.

ADAPTIVE CONTROL SCHEME

Model Reference

The first step in designing an adaptive control system is to define a figure of merit. This can be done by considering the basic plant vs. model system shown in Figure 2, where the model can be chosen at the discretion of the system designer. It is desired to force the plant to behave as much as possible like the model, so an error signal is defined as

$$e(t) = x(t) - x_m(t) \tag{1}$$

The purpose of the adaptive control system is to minimize this error over time with respect to a plant parameter λ_k. The error criterion that will be used is integral-error-squared.

The integral-error-squared, or performance index PI, is

$$PI = \int_o^t e^2(t)\,dt \tag{2}$$

The curve of performance index vs. λ_k should exhibit a minimum for each of the plant parameters if all the other parameters are held constant. However, because of the changing conditions of the system and surroundings, the curve itself can be considered an unknown function of time. Thus, knowledge of the value of PI or λ_k at any instant does not yield any useful information as to how to adjust λ_k to minimize PI. On the other hand, the slope of the curve $\partial PI/\partial\lambda_k$ would be very useful. That is, if $\partial PI/\partial\lambda_k$ can be periodically determined, then λ_k could be adjusted and PI kept near its minimum value.

Development of Equations

In order to calculate $\partial PI/\partial\lambda_k$ periodically, the following manipulations are necessary. Given

$$PI = \int_o^t e^2(t)\,dt \tag{2}$$

take partials with respect to λ_k

$$\frac{\partial PI}{\partial\lambda_k} = \int_o^t \frac{\partial e^2(t)}{\partial\lambda_k}\,dt = 2\int_o^t e(t)\,\frac{\partial e(t)}{\partial\lambda_k}\,dt \tag{3}$$

But

$$e(t) = x(t) - x_m(t) \tag{1}$$

Therefore

$$\frac{\partial e(t)}{\partial\lambda_k} = \frac{\partial x(t)}{\partial\lambda_k} - \frac{\partial x_m(t)}{\partial\lambda_k} \tag{4}$$

However

$$\frac{\partial x_m(t)}{\partial\lambda_k} = 0 \tag{5}$$

and

$$\frac{\partial PI}{\partial\lambda_k} = 2\int_o^t e(t)\,\frac{\partial x(t)}{\partial\lambda_k}\,dt \tag{6}$$

Thus, if $\partial x(t)/\partial\lambda_k$ can be determined, with $e(t)$ measured, the slope of the performance index curve can also be determined. Then, on the basis of this value of the slope, λ_k can be modified in a prescribed manner in order to minimize the performance index for the system at that instant.

Objective of Adaptivity

It must be kept in mind that the objective of the adaptive control system is to keep the response of $x(t)$ to changes in an input signal $x_i(t)$ as close to a specified response as possible in the presence of various disturbances. The relationships between plant output and other inputs, such as cooling water temperatures, are not the object of any control effort, although these other variables have to be taken into account in order to achieve the primary objective. By keeping the dynamic relationship between $x(t)$ and $x_i(t)$ constant, one can insure that the plant will always react to changes in $x_i(t)$ at any stage during system operation with the specified dynamic performance.

REACTING SYSTEM

Figure 3 is a schematic diagram of the continuous stirred tank reactor. The data for the reactor are the same

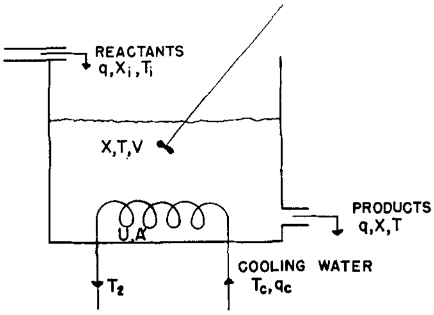

Fig. 3. Schematic reactor.

as those used by Kermode and Stevens (8). The principal assumptions are that there is a constant volume in the reactor and that the reaction rate constants are governed by the Arrhenius equation

$$k = A\epsilon^{-E/RT} \tag{7}$$

The reaction involved is a simple first-order, unidirectional, exothermic decomposition

$$\text{Reactant} \xrightarrow{-\Delta H} \text{Products}$$

Taking a mass balance on the reactant

$$qx_i = qx + kxV + V\frac{dx}{dt}$$

or

$$\frac{dx}{dt} = \frac{q}{V}(x_i - x) - kx \tag{8}$$

The heat balance is

$$\rho c_p q T_i = \rho c_p q T - kx(-\Delta H)V + \rho c_p V\frac{dT}{dt} + UA'\Delta T_m$$

or

$$\frac{dT}{dt} = \frac{q}{V}(T_i - T) + \frac{kx(-\Delta H)}{\rho c_p} - \frac{UA'\Delta T_m}{V\rho c_p} \tag{9}$$

where ΔT_m comes from a heat balance on the cooling coil

$$UA'\Delta T_m = q_c\rho_c c_c\,(T_2 - T_c). \tag{10}$$

ΔT_m is assumed to be the arithmetic mean temperature difference between reactor contents and cooling coil. This leads to an expression for ΔT_m, which is

$$\Delta T_m = \frac{T - T_c}{1 + \dfrac{1}{F}} \tag{11}$$

where

$$F = \frac{UA'}{2}\,q_c\rho_c c_c$$

Equations (8) and (9) form the set of simultaneous nonlinear differential equations which can be used to represent the continuous stirred tank reactor. The only variation necessary in the equations to make them represent the closed-loop plant is simply to include the additional restriction placed on the cooling water flow rate by the controller. That is

$$q_c = q_{cs} + G_c\,(x - x_s) \tag{12}$$

Linearization

Since the calculation of $\partial PI/\partial\lambda_k$ requires a relationship between $x(t)$ and λ_k, it is desirable to simplify Equations (8) and (9) by linearizing about the steady state operating point of the system. This introduces the restriction to the subsequent operation of the reactor, that the reactor must stay in the region for which the linearized equations are valid.

After expanding in Taylor's series about the steady state operating point and simplifying, the nonlinear terms become

$$kx \doteq k_s x_s + k_s\,(x - x_s) + \frac{x_s E k_s}{RT_s^2}(T - T_s) \tag{13}$$

and

$$\Delta T_m \doteq \Delta T_{ms} + \frac{2q_{cs}\rho_c c_c}{2q_{cs}\rho_c c_c + UA'}(T - T_s) + \frac{UA'\,(T_s - T_c)}{2q^2_{cs}\rho_c c_c\left(1 + \dfrac{1}{F}\right)^2}(q_c - q_{cs}) \tag{14}$$

Thus, the linear system of equations that approximates the reactor at steady state is

$$\frac{d\bar{T}}{dt} = \frac{k_s(-\Delta H)}{\rho c_p}\bar{x} + \left[\frac{x_s E k_s(-\Delta H)}{RT_s^2\rho c_p} - \frac{q}{V} - \frac{UA'(2q_{cs}\rho_c c_c)}{V\rho c_p(2q_{cs}\rho_c c_c + UA')}\right]\bar{T} - \frac{U^2A'^2\,(T_s - T_c)}{V\rho c_p(2q^2_{cs}\rho_c c_c)\left(1 + \dfrac{1}{F}\right)^2}\bar{q}_c \tag{15}$$

$$\frac{d\bar{x}}{dt} = \frac{q}{V}\bar{x}_i - \left[\frac{q}{V} + k_s\right]\bar{x} - \frac{x_s E k_s}{RT_s^2}\bar{T} \tag{16}$$

For simplicity's sake

$$\frac{d\bar{T}}{dt} + K_1\bar{T} = K_2\bar{x} - K_3\bar{q}_c \tag{17}$$

$$\frac{d\bar{x}}{dt} + K_4\bar{x} = K_5\bar{x}_i - K_6\bar{T} \tag{18}$$

The use of Laplace transforms (using the fact that the system is initially at steady state) and the substitution of Equation (17) into Equation (18) results in

$$X(s) = \frac{K_5(s + K_1)}{[(s + K_1)(s + K_4) + K_2K_6]}X_i(s) + \frac{K_3K_6}{[(s + K_1)(s + K_4) + K_2K_6]}\,Q_c(s) \tag{19}$$

Assuming proportional control only

$$G_c(s) = K_c \tag{20}$$

and closing the loop on the ordinary feedback control system within the plant block

$$\frac{X(s)}{X_i(s)} = \frac{K_5(s + K_1)}{[s^2 + (K_1 + K_4)s + K_1K_4 + K_2K_6 + K_cK_3K_6]} \tag{21}$$

In like manner, the relationships between $X(s)$ and any other plant inputs, such as $Q_c(s)$, could be determined.

APPLICATION OF ADAPTIVE SCHEME

The first problem in the application of the adaptive scheme to this reacting system is the choice of the model. In this case the model was logically, though not necessarily, chosen to be the closed-loop plant transfer function at its steady state operating point. That is

$$\frac{X_m(s)}{X_i(s)} = \frac{K_5(s + K_1)}{s^2 + (K_1 + K_4)s + (K_1K_4 + K_2K_6 + K_cK_3K_6)} \tag{22}$$

Thus, $e(t)$ differs from zero only if the plant parameters imbedded in the various coefficients differ from their steady state values.

It now becomes the job of the adaptive control system to drive the plant to approximate the performance of Equation (22), whose denominator in standard notation can be written as

$$s^2 + 2\zeta\omega s + \omega^2 = s^2 + (K_1 + K_4)s + (K_1K_4 + K_2K_6 + K_cK_3K_6) \tag{23}$$

Using the parameter values from Kermode and Stevens, one can see that

$$2\zeta\omega = K_1 + K_4 = 8.65 \times 10^{-3} \tag{24}$$

and

$$\omega^2 = K_1K_4 + K_2K_6 + K_cK_3K_6 = 116.65 \times 10^{-6} \tag{25}$$

where the initial value of K_c is 10.8. Therefore, ω = 10.71×10^{-3} cycles/sec. and $\zeta = 0.40$.

The next problem is the determination of $\partial PI/\partial\lambda_k$ for this reacting plant. For the plant

$$\frac{X(s)}{X_i(s)} = \frac{K_5(s + K_1)}{s^2 + (K_1 + K_4)s + (K_1K_4 + K_2K_6 + K_cK_3K_6)} \tag{26}$$

Taking the derivative of the transfer function and using K_c as λ_k

$$\frac{\partial X(s)}{\partial K_c} = \frac{-K_3K_6X(s)}{s^2 + (K_1 + K_4)s + K_1K_4 + K_2K_6 + K_cK_3K_6} \tag{27}$$

or

$$\frac{\partial X(s)}{\partial K_c} = \left[\frac{-K_3K_6}{K_5(s + K_1)}\right] \cdot \left[\frac{X(s)}{X_i(s)}\right] \cdot X(s) \tag{28}$$

This equation can be rewritten as

$$\frac{\partial X(s)/\partial K_c}{X(s)} = \frac{-K_3K_6}{K_5(s + K_1)} \cdot \frac{X(s)}{X_i(s)} \tag{29}$$

Upon inversion Equation (29) becomes a relationship between $\partial x(t)/\partial K_c$ and $x(t)$. As described above that relationship can be used to calculate

$$\frac{\partial PI}{\partial K_c} = \int_0^t e(t) \cdot \frac{\partial x(t)}{\partial K_c}\, dt \tag{30}$$

Identification Scheme

At this point all the blocks shown schematically in Figure 4 are defined, except for the decision block and the block containing Equation (29) inverted. In order to find Equation (29), however, an automatic identification method must be incorporated into the control system, since $X(s)/X_i(s)$ is unknown. There are many methods available but the one chosen was that of Kalman (9).

In this method the input and output signals of the plant are sampled at set intervals and are then correlated to determine the a_i and the b_j in the following second-order, pulse-transfer function

$$\frac{X(z)}{X_i(z)} = \frac{a_1z^{-1} + a_2z^{-2}}{1 + b_1z^{-1} + b_2z^{-2}} \tag{31}$$

This approximate pulse-transfer function can be used to help define Equation (29). However, use of a pulse-transfer function necessitates simulation of the system in terms of discrete quantities. Thus, Equation (29) becomes, in the z transform notation

$$\frac{\partial X(z)/\partial K_c}{X(z)} = \frac{\frac{K_6}{K_5}\tau z^{-1}}{1 + (K_1\tau - 1)z^{-1}} \cdot \frac{X(z)}{X_i(z)} \tag{32}$$

Equation (32) can easily be inverted into a difference equation. Figure 4 can then be transformed into Figure 5, which is the schematic block diagram of the overall adaptive control system in terms of discrete variables.

Kalman's identification method is a relatively fast and precise one, but it has two disadvantages: It requires that a small test signal be superimposed on the input signal $x_i(t)$; and being discrete, it necessitates treatment of the system as a sampled data system.

SIMULATION

The overall discrete signal block diagram of Figure 5 was simulated on a digital computer in the following manner. The signals $\overline{x}_i(t)$ and the other disturbances were generated continuously in the form described below, and the signal $\overline{x}(t)$ was determined by the simultaneous numerical solution of the set of nonlinear differential equa-

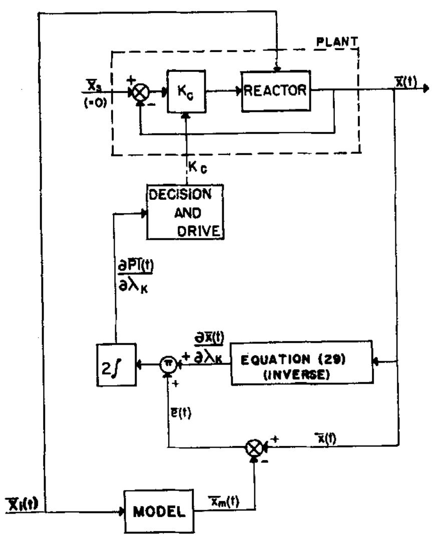

Fig. 4. Continuous overall system.

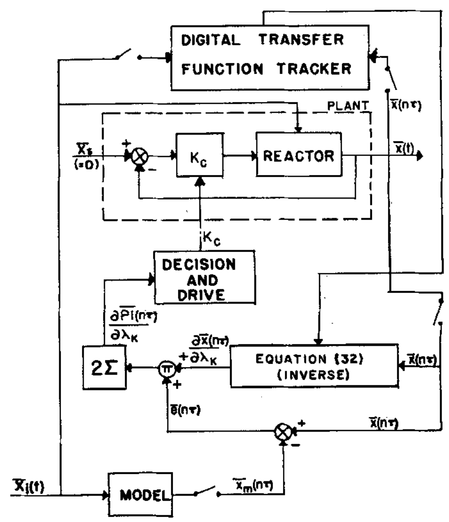

Fig. 5. Discrete overall system.

tions (8) and (9), remembering that $\overline{x}(t) = x(t) - x_s$. At each instant $n\tau (n = 1,2,\ldots)$, the values of $\overline{x_i}(n\tau)$, $\overline{x}(n\tau)$, and the other disturbances were sampled. On the basis of these sampled values of $\overline{x_i}(n\tau)$ and $\overline{x}(n\tau)$, a current updated approximate pulse-transfer function for the plant $\overline{X}(z)/\overline{X_i}(z)$ was determined by the Digital Transfer Function Tracker for use in Equation (32).

At the same time $\overline{x_i}(t)$ was input to the model block to calculate $\overline{x_m}(t)$, using Equation (22) inverted. The sampled values of $\overline{x_m}(n\tau)$ and $\overline{x}(n\tau)$ were then used to determine the values of $\partial \overline{x}(n\tau)/\partial\lambda_k$, $\overline{e}(n\tau)$ and $\partial \overline{PI}(n\tau)/\partial\lambda_k$, as shown in Figure 5. This last value was then the basis for the specification of a new value of λ_k, using the fact that, initially, $\partial PI/\partial K_c = 0$. By repeating this process at each sampling instant, K_c was kept near its optimal value.

Two different quantities were used as disturbances: the cooling water inlet temperature and a catalyst activity. The temperature disturbance was applied in the forms of a sinusoidal and a random disturbance, each form having a mean at the steady state value. The catalyst activity was assumed to affect the rate constants by appearing as a factor in the Arrhenius equation

$$k = aA\epsilon^{-E/RT} \tag{33}$$

The catalyst disturbance was applied as a straight-line decay and as an exponential decay.

Start-up problems that appeared in the calculations were eliminated by simulating the reactor operation at steady state for 5 min. before applying any disturbances. As can be seen in Table 1, eight cases were run with a cooling water temperature disturbance alone, six with a catalyst activity decay only, and one with both temperature and activity disturbances together.

DECISION PROCESSES

Several different decision processes were investigated with this adaptive scheme, one of which was a modified Newton-Raphson root-seeking method, with the objective being the adjustment of the feedback gain in an optimal manner. While all the decision rules gave reasonably good results, the modified Newton-Raphson method seemed the best (*10*).

The Newton-Raphson equation is (*11*)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{34}$$

Sequential application of this equation to a function will determine the roots of the function. If the function is considered to be

$$f(K_c) = \frac{\partial PI}{\partial K_c} \tag{35}$$

the equation becomes

$$K_{c_{n+1}} = K_{c_n} - \frac{f(K_{c_n})}{f'(K_{c_n})} \tag{36}$$

The $f(K_{c_n})$ has already been determined, but

$$f'(K_{c_n}) = \frac{\partial^2 PI}{\partial K_c^2}$$

still must be derived. With the same procedure as was used for deriving the first partial, it follows from Equation (6) that

$$\frac{\partial^2 PI}{\partial\lambda_k^2} = 2\int_0^t \left[\frac{\partial x(t)}{\partial\lambda_k}\right]^2 dt + 2\int_0^t e(t)\cdot\frac{\partial^2 x(t)}{\partial\lambda_k^2}dt \tag{37}$$

The former integral can be determined, since $\partial x(t)/\partial\lambda_k$ is already known. However, the second integral can be evaluated only by determining $\partial^2 x(t)/\partial\lambda_k^2$. Taking the derivative of Equation (28)

$$\frac{\partial^2 X(s)}{\partial K_c^2} = \left[\frac{-2(\partial X(s)/\partial K_c)(K_3K_6)}{K_5(s+K_7)}\right]\cdot\left[\frac{X(s)}{X_i(s)}\right] \tag{38}$$

Upon inversion, this is the desired expression.

In order to obtain more rapid and precise convergence, Lance (*12*) developed a modification of the Newton-Raphson formula. The ordinary equation, (34), is applied once to the system, and if

$$|f(K'_{c_{n+1}})| < |f(K_{c_n})|$$

it means that an improvement has been obtained. Therefore, the sequence of values

$$f\left[K_{c_n} - \frac{nf(K_{c_n})}{f'(K_{c_n})}\right],\ n = 2,3,4,\ldots, N$$

is computed until a value N of n is reached, where an improvement in subsequent values is no longer seen. Thus, leaving the iteration

$$K_{c_{n+1}} = K_{c_n} - \frac{Nf(K_{c_n})}{f'(K_{c_n})} \tag{39}$$

TABLE 1. TABULATION OF RESULTS

Case No.	Disturbance*	Adapted system I.E.S.,† %	Time t, sec.
1	Random $\overline{T_c}$ of ±0.5°F. (period of 10 min.)	5.43	2,400
2	Some as case 1, with Δx_i = ±0.0015 lb./cu. ft.	21.9	1,200
3	Sinusoidal $\overline{T_c}$ with ampl. = ±10°F. and per = 20 min.	6.67	2,400
4	Same as case 3, with Δx_i = ±0.0015 lb./cu. ft.	13.8	3,400
5	Sinusoidal $\overline{T_c}$ with ampl. = ±10°F. and per = 10 min.	6.98	2,400
6	Sinusoidal $\overline{T_c}$ with ampl. = ±15°F. and per = 20 min.	6.78	3,000
7	Sinusoidal $\overline{T_c}$ with ampl. = ±20°F. and per = 20 min.	6.07	3,000
8	Sinusoidal $\overline{T_c}$ with ampl. = ±20°F. and per = 10 min.	24.8	1,600
9	Activity decay of 10% a_o each 20 min.	1.47	2,400
10	Activity decay of 20% a_o each 20 min.	1.50	2,400
11	Activity decay of 10% a_o first 20 min.; a constant thereafter	1.95	2,400
12	Exponential activity decay of $a = a_o e^{0.000186t}$	0.71	2,200
13	Activity decay of 20% a_o each 20 min.	8.12	4,400
14	Activity decay of 20% a_o each 20 min.	0.40 (with steady state operation for 5 min.)	4,000
15	Sinusoidal T_c with ampl. = ±15°F. and per = 10 min., and an activity decay of 20% a_o each 20 min.	0.56	4,800

*Unless otherwise noted, in each case the superimposed disturbance on input composition is a random ± 0.0005 lb./cu. ft. applied every 60 sec. for 30 sec. and zero for the remaining 30 sec.
† Errors expressed as a percentage of the unadapted system error at time t.

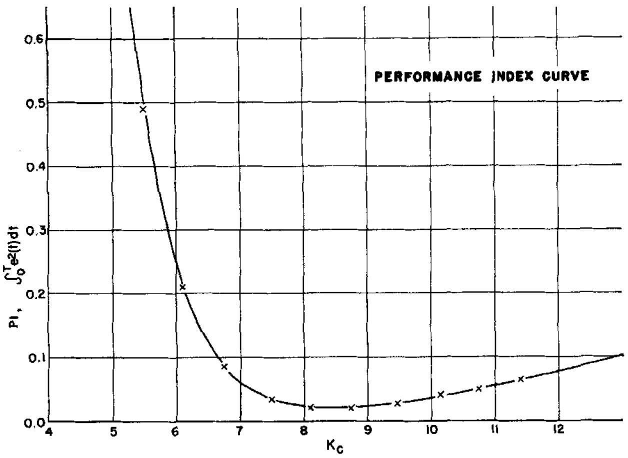

Fig. 6. Performance index curve.

In like manner, if

$$|f(K'_{c_{n+1}})| \geq |f(K_{c_n})|$$

it means that the step in K_c was too large, so that the sequence of values

$$f\left[K_{c_n} - \frac{f(K_{c_n})}{2^n f'(K_{c_n})}\right], \ n = 2,3,4,\ldots, N$$

is computed until no more improvement is shown. Then

$$K_{c_{n+1}} = K_{c_n} - \frac{f(K_{c_n})}{2^N f'(K_{c_n})} \qquad (40)$$

RESULTS

Performance Index Curve

As a first step toward the mechanization of the adaptive control system, an idea of the general nature of the performance index curve had to be determined. The curve that was obtained for step input in $x_i(t)$ of 0.02 lb./cu. ft. is shown in Figure 6. As can be seen the curve does exhibit a definite minimum, at about a value of K_c of 9.0. The curve is not symmetrical about the minimum point, which was expected, since an overdamped system (large K_c) will deviate from a specified behavior much more slowly than will an underdamped system (small K_c). It could have made for an easier decision process had the curve been symmetrical, but the fact that it was not did not prove too detrimental.

Discrete Signal Data

As mentioned above, the inclusion of Kalman's identification scheme necessitated treatment of the system by discrete signals. The sampling time used was 10 sec. and the adjustment time (of feedback gain) was 30 sec. Using this sampling period, the pulse-transfer function for the plant at steady state became

$$\frac{X(z)}{X_i(z)} = \frac{0.050\,z^{-1} - 0.048\,z^{-2}}{1 - 1.913\,z^{-1} + 0.924\,z^{-2}} \qquad (41)$$

Adaptive vs. Nonadaptive System Results

The overall adaptive control system was simulated for a variety of disturbances, and the integral-error-squared was calculated for each case for at least 1/3 hr. of reactor operating time. Furthermore, the integral-error-squared between the model and the ordinary feedback system output was calculated for the same disturbances, and was compared with that for the corresponding adapted case.

As can be seen for the cases where the larger test signal was used, adaptivity was not as beneficial as it could have been. This is exhibited by the results showing the I.E.S. for cases 1 and 3 being considerably less than those for cases 2 and 4. In all situations where the test signal of ± 0.0005 lb./cu. ft. was used, the results were comparable, except for case 8. Case 8 had the largest amplitude sinusoidal temperature disturbance combined with the highest frequency used, yielding less satisfactory results.

For the remaining cases of a sinusoidal temperature disturbance alone, the average adapted system I.E.S. expressed as a percentage of the unadapted system I.E.S. was 6.63%, the maximum deviation being 0.56% for case 7.

For the cases where there was only a catalyst activity disturbance, the worst adapted system I.E.S. of 1.95% of the unadapted system I.E.S. at 2,400 sec. occurred for a straight-line activity decay of 10% of the initial activity for 20 min. (case 11). As expected, the percentage error generally decreased for a faster rate of activity decay, the best result being obtained where the decay was exponential. In all of the cases simulated, the unadapted system integral-error-squared was invariably much greater than that for the adaptive system.

To prove the number of disturbances is unimportant, the overall system was simulated for multidisturbances. The disturbances, shown in Figure 7, were a sinusoidal cooling water temperature disturbance of ± 15°F. and a period of 600 sec., and a catalyst activity decay of 20% of its initial value every 20 min. The results for this case are shown in Figures 8 and 9. As can be seen, the adapted system I.E.S. was 0.56% of the unadapted I.E.S. at 4,800 sec.

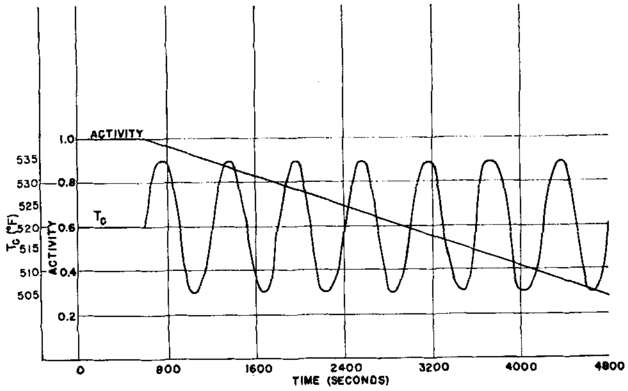

Fig. 7. Disturbances vs. time for case 15.

CONCLUSIONS

The method presented here for adaptive control of a closed-loop plant is applicable universally. Several refine-

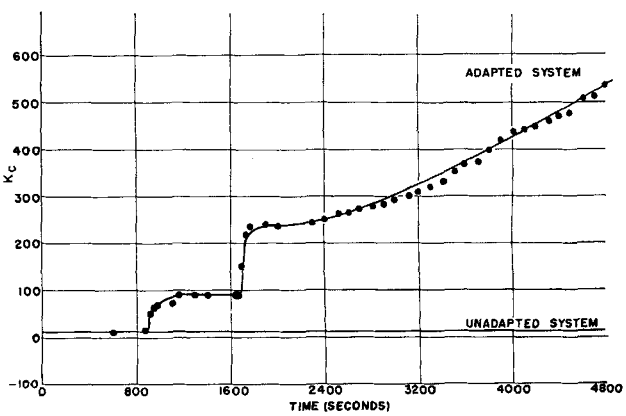

Fig. 8. Feedback gain vs. time for case 15.

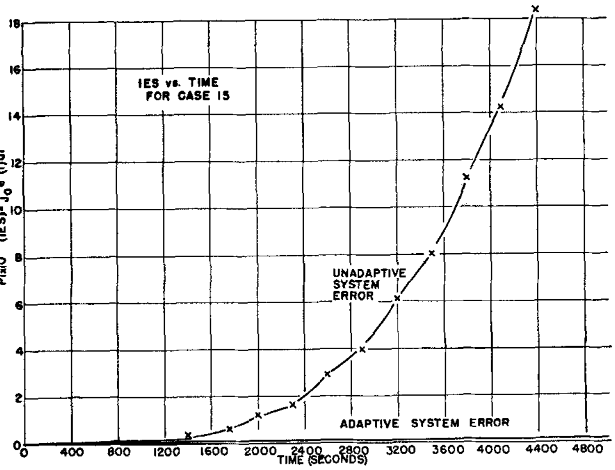

Fig. 9. I.E.S. vs. time for case 15.

ments could be made in the calculation scheme, but the results obtained in this work were quite satisfactory.

However, some of the limitations of the method should be pointed out. Most important, the fact that the controller was a proportional element only tended to bias the results in favor of the adaptive system over the unadaptive system. In light of this the next item of study should be the comparison of performance using a proportional-plus-integral, or a proportional-plus-integral-plus-derivative controller. It is suspected that the improvement in performance due to adaptivity won't be anywhere near as dramatic.

Several limitations were introduced by the use of Kalman's identification scheme. The necessity of using discrete, rather than continuous, measurement of the continuous variables can lead to serious error if the variables are rapidly changing. Thus, an assumption implicit in the work was that the variables are slowly varying with respect to the sampling interval. Kalman's method also necessitated use of a small test disturbance on the input signal x_i. These difficulties could be eliminated by the development of a better identification scheme.

Another assumption which should be emphasized is that there was no "noise" in the system. This does not limit the adaptive scheme mathematically, but the effects of noise on the system must be investigated before mechanization.

Thus, it would seem that the inclusion of an adaptive control capability as presented here should be investigated whenever it seems to be possible physically. However, the final factor to be considered must be economic feasibility.

NOTATION

a = catalyst activity
A = frequency factor in Arrhenius equation, (sec.$^{-1}$)
A' = heat transfer area of reactor cooling coil, sq. ft.
a_i = coefficient of z^{-i} term in numerator of plant pulse transfer function
b_j = coefficient of z^{-j} term in denominator of plant pulse transfer function
c_c = heat capacity of coolant, B.t.u./(lb.) (°R.)
c_p = heat capacity of reactor contents, B.t.u./(lb.) (°R.)
e = difference between model output x_m and plant output x (lb.-mole) (cu. ft.)
E = molal energy of activation for reaction, B.t.u./lb.-mole
$G_c(s)$ = controller transfer function
ΔH = heat of reaction, B.t.u./lb.-mole
$I.E.S.$ = integral-error-squared
K_c = proportional feedback gain
k = reaction rate constant, sec.$^{-1}$
PI = performance index
q = flow rate of reactor inlet and outlet streams, cu. ft./hr.
q_c = flow rate of coolant, cu. ft./hr.
$Q_c(s)$ = Laplace transform of $\bar{q}_c$
R = gas constant, B.t.u./(lb.-mole) (°R.)
t = time, sec.
T = temperature of reactor contents, °R.
$T(s)$ = Laplace transform of $\bar{T}$
T_c = temperature of coolant, °R.
$T_c(s)$ = Laplace transform of $\bar{T}_c$
T_i = temperature of reactor inlet stream, °R.
ΔT_m = arithmetic mean temperature difference between reactor contents and cooling coil, °R.
T_2 = outlet temperature of coolant, °R.
U = heat transfer coefficient between reactor contents and cooling coil, B.t.u./(hr.-sq. ft.) (°R.)
V = volume of reactor contents, cu. ft.
x = reactant composition in reactor outlet stream, lb.-mole/cu. ft.
$X(s)$ = Laplace transform of $\bar{x}$
$X(z)$ = pulse transform of $\bar{x}$
x_i = reactant composition in reactor inlet stream, lb.-mole/cu. ft.
$X_i(s)$ = Laplace transform of $\bar{x}_i$
$X_i(z)$ = pulse transform of $\bar{x}_i$
x_m = model output composition, lb.-mole/cu. ft.
$X_m(s)$ = Laplace transform of $\bar{x}_m$
Z = activating signal from computer

Greek Letters

ζ = damping ratio
λ = plant parameter
ρ = density of reactor contents, lb./cu. ft.
ρ_c = density of coolant, lb./cu. ft.
τ = sampling period, sec.
ω = frequency, cycles/sec.

Subscript

s = steady state value

Superscript

— = deviation from steady state value

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